Chapter - 12

Areas Related to Circles

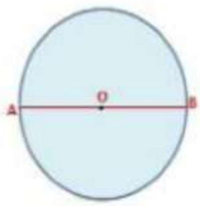
Perimeter and Area of a Circle

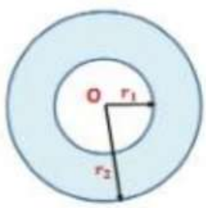
Introduction

We come across many objects in our life that is circular like,
- cycle wheels
- dartboard
- doughnut
- bangles
- wheelbarrow

A circle is a collection of all points in a plane that are at a constant distance (radius) from the fixed point (center).

O	Center of the Circle
OA	Radius of the Circle
AB	Diameter of the Circle





1. Diameter of a circle is twice the radius.
2. Two or more circles having the same center are called concentric circles.

Perimeter and Area of a Circle
Perimeter is defined as the distance around a closed figure. In the case of a

circle, the perimeter of a circle is called its circumference. The ratio of the circumference and diameter of a circle is constant. This, constant ratio is denoted by the Greek letter π.

$$\frac{\text{Circumference}}{\text{diameter}} = \pi \Rightarrow \text{Circumference} = \pi \times \text{diameter}$$

Circumference $= \pi \times 2r$, wherer is the radius of the circle.

The great Indian Mathematician Aryabhatta (476 – 550 AD) gave the approximate value of π.

$\pi = \frac{62832}{20000} \approx$ **3.14.** For practical purpose we take the value of π as $\frac{22}{7}$ or 3.14 .

Area of a circle is πr^2, where r is the radius of the circle. We can verify this by cutting a circle into several sectors and rearranging them as shown below:

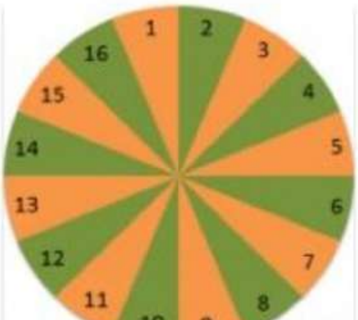


Fig 1

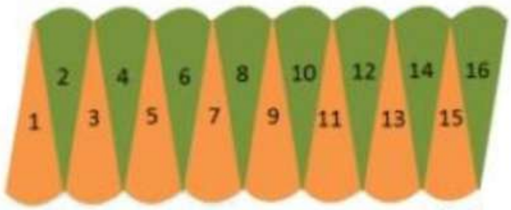


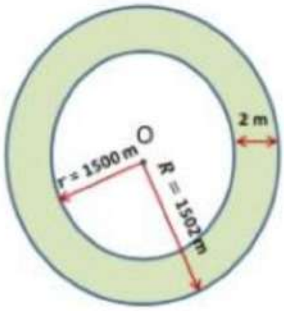
Fig 2

Now, the shape in Fig 2 is nearly a rectangle of length $\frac{1}{2} \times 2\pi r$ and the breadth of this rectangle is equal to the radius of the circle 'r'.

Area of the circle = Area of the rectangle formed = $l \times b = 1/2 \times 2\pi r \times r = \pi r^2$

Example: Circular footpath of width 2 m is constructed at the rate of Rs. 10 per m^2 around a circular park of radius 1500 m. Find the total cost of the construction of the path.

[Take $\pi = 3.14$]



Let r = 1500 m be the inner radius of the circle.

Width of the footpath = 2 m

Therefore, outer radius of the circle R = (1500 + 2) m

Area of the footpath $= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2) = 3.14(15022 - 1500^2)$

$= 3.14(1502 + 1500)(1502 - 1500)$

$\{\because a^2 - b^2 = (a + b)(a - b)\}$

$= 3.14(3002 \times 2) = 3.14 \times 6004 = 18852.6 \text{ m}^2$

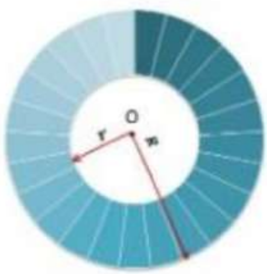
The total cost of construction of the path $= 10 \times 18852.6$

= Rs 188526

Example: The shaded area in the adjacent figure between the circumference of two concentric circles is 330 cm^2. The circumference of the inner circle is 88 cm. Calculate the radius of the outer circle.

Let r be the radius of the inner circle and R be the radius of the outer circle.

We know that circumference of the inner circle = 88 cm



$2\pi r = 88 \Rightarrow 2 \times \frac{22}{7} \times r = 88 \Rightarrow r = \frac{88X7}{44} = 14$ cm

Area of the shaded region $= \pi R^2 - \pi r^2$

$330 = \frac{22}{7} \times (R^2 - 14^2)$

$330 \times 722 = R^2 - 14^2 \Rightarrow 105 + 196 = R^2$

$R^2 = 301 \Rightarrow R = 17.35$ cm

The radius of the outer circle is 17. 35 cm

Example: The diameter of a cycle wheel is 14 cm. How many revolutions will it make to travel 2.64 km?

Diameter of the cycle wheel = 14 cm

Radius of the wheel of the car $= \frac{14}{2}$ cm $= 7$ cm

Circumference of the wheel $= 2\pi r = 2 \times \frac{22}{7} \times 7 = 44$ cm

Distance covered by the wheel = 2. 64 km

$= 2.64 \times 1000 \times 100$ cm

Number of revolutions $= \frac{Distance\ covered\ by\ the\ wheel}{Circumference\ of\ the\ wheel}$

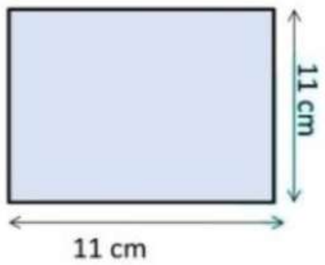
$= \frac{2.64X1000X100}{44} = 6000$

Example: A wire when bent in the form of a square encloses an area 121 cm^2. If the wire was bent in the form of a circle, then find the area enclosed by the circle.

The wire is bent in the form of a square.

We know, area of a square = $(\text{side})^2 = 121\ \text{cm}^2$

$\therefore (\text{side})^2 = (11\ \text{cm})^2 \Rightarrow \text{side} = 11\ \text{cm}$

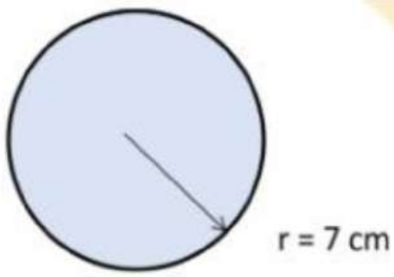


Perimeter of the square = $4 \times \text{side} = 4 \times 11 = 44$ cm

Now if the same wire is bent in the form of a circle then the perimeter of the square is equal to the circumference of the circle.

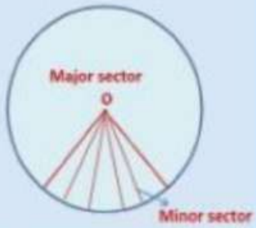
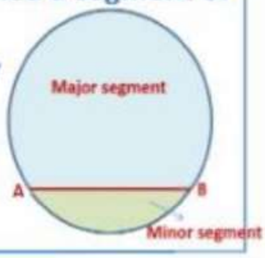
$\therefore$ Circumference of the circle = $2\pi r = 44$ cm

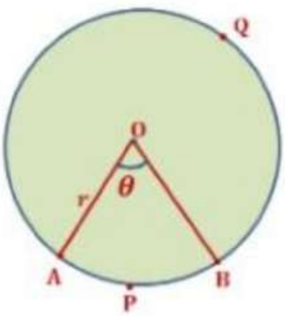
$\Rightarrow 2 \times \frac{22}{7} \times r = 44\ \text{cm} \Rightarrow r = \frac{44X74}{4} = 7\ \text{cm}$



Area of Sector and Segment of a Circle

Areas of Sector and Segment of a circle

<table>
<tr>
<td>Sector</td>
<td>The region between an arc and the two radii, joining the ends of the arc to the center, is called a sector.
Minor Sector – The sector formed by the minor arc.
Major Sector – The sector formed by the major arc

</td>
</tr>
<tr>
<td>Segment</td>
<td>The region between a chord and either of its arcs is called a segment of the circle.
Minor Segment – The segment formed between minor arc and the chord.
Major Segment - The segment formed between major arc and the chord.

</td>
</tr>
</table>



Let OAPB be a sector of the circle with center O and radius. Let the degree of measure of ∠AOB be θ.

We know that the area of a circle = πr^2

Now this circular region can be considered to be a sector forming an angle of 360° at the centre O.

We now apply the unitary method to find the area of the sector APB.

When the degree measure of the angle at the centre is 360, area of the sector = πr^2

So, when the degree measure of the angle at the centre is 1, area of the sector

$$= \frac{\pi r^2}{360}$$

Therefore, when the degree measure of the angle at the centre is θ, area of the sector $= \frac{\pi r^2}{360} \times \theta = \frac{\varnothing}{360} \times \pi r^2$

Area of the sector of angle $\theta = \frac{\varnothing}{360} \times \pi r^2$ where r is the radius of the circle and θ the angle of the sector in degrees.

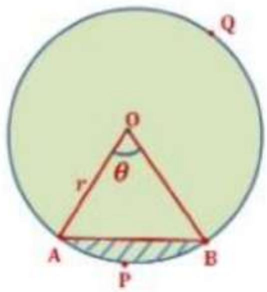
Similarly, by applying the Unitary Method and taking the whole length of the circle as 2πr, we can obtain the required length of the arc APB as $\frac{\varnothing}{360} \times 2\pi r$

Length of an arc of a sector of angle $\theta = \frac{\varnothing}{360} \times 2\pi r$

Area of the segment APB = Area of sector OAPB – Area of Δ AOB

$= \frac{\varnothing}{360} \times \pi r^2 -$ Area of Δ AOB

Therefore,



Area of the major sector OAQB

$= \pi r^2$- Area of the minor sector OAPB

Area of the major segment AQB = πr^2 - Area of the minor sector APB

Example: The minute hand of the clock is 7 cm. Find the area of the face of the clock described by the minute hand between 9 am and 9:30 am.

Angle described by the minute hand in 60 min = 360°

∴ Angle described by the minute hand in 1 min = $\frac{360^\circ}{60} = 6^\circ$

Now, the angle described by minute hand in 30 min

$= 30 \times 6^{o} = 180^{o}$

Area swept by the minute hand in 30 min

= Area of the sector of angle 180^{o} and radius 7 cm

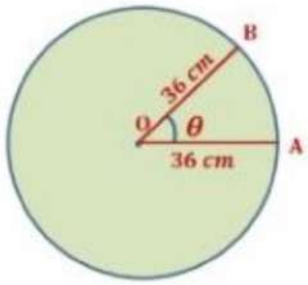
We know, area of the sector of angle $\theta = \frac{ø}{360} \times \pi r^2$

Area of the sector $= \frac{180}{360} \times \frac{22}{7} \times (7)^2$

$= 77\ cm^2$

Example: Area of a sector of a circle of radius 36 cm is $72\pi\ cm^2$.

Find the length of the corresponding arc of the sector.



Radius of circle = 36 cm

Area of a sector of the circle $= 72\pi\ cm^2$

Area of the sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

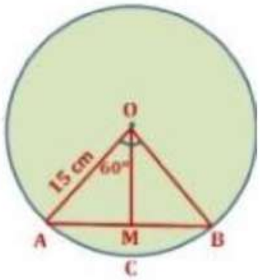
$72\pi = \frac{\theta}{360} \times \pi(36)^2 \Rightarrow \frac{\theta}{360} = \frac{72\pi}{\pi(26)^2} = \frac{1}{18}$

We know, Length of an arc of the sector of angle $\theta = \frac{\theta}{360} X 2\pi r$

Length of an arc of the sector = $\frac{1}{18} X 2 X \pi X 36 = 4\pi cm$

Example: A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. [Take π = 3.14 and √3 = 1.73]

(REFERENCE: NCERT)



Here, chord AB subtends an angle of 60o at the center.
$\angle$ AOB = 60°
OA = OB (radii of the circle)
∴ $\angle$OAB = $\angle$OBA = x (Angles opposite to equal sides are equal)
In Δ AOB,
$\angle$ AOB + $\angle$OAB +$\angle$OBA = 180°
60° + x + x = 180° ⟹ 60° + 2x = 180° ⟹ 2x = 120°

x = 60^{o}

$\angle$ AOB = $\angle$OAB =$\angle$OBA = 60^{o}

∴Δ AOB is an equilateral triangle

Area of Δ AOB = $\frac{\sqrt{3}}{4} \times (15)^2 = 56.25 \times 1.73 = 97.3125$ cm^2

Area of sector OACB = $\frac{60}{360} \times 3.14 \times (15)^2 = 117.75$ cm^2

Area of minor segment ACB = Area of sector OACB - Area of Δ AOB

= 117.75 cm^2 − 97.3125 cm^2 = 20.4375 cm^2

The area of the corresponding major sector

$= \text{Area of the circle} - \text{Area of minor segment} = \pi r^2 - 20.4375$

$= 3.14 \times (15)^2 - 20.4375 = 706.5 - 20.4375$

$= 686.0625 cm^2$

Example: An umbrella has 8 ribs that are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

(REFERENCE: NCERT)

Central Angle of the umbrella is 360°.

Now, the umbrella has 8 ribs.

$\therefore$ Angle between two ribs $= \dfrac{360}{8} = 45°$

Area between two ribs = Area of one sector of the umbrella $= \dfrac{\theta}{360} \times \pi r^2$

Area of the sector $= \dfrac{45}{360} X \dfrac{22}{7} X (45)^2 = \dfrac{22275}{28} cm^2$

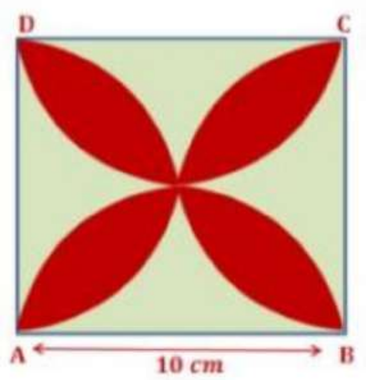
Area of Combination of Plane Figures

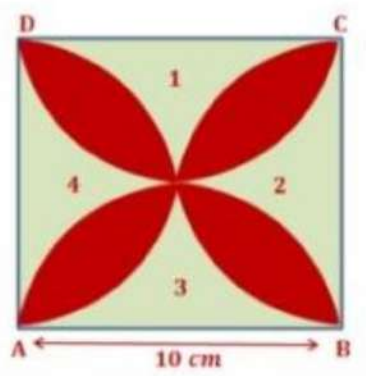
Areas of Combination of Plane Figures

We come many figures in our daily life which are combinations of plane figures and form various interesting designs like flower beds, drain covers, designs on table covers, circular paths. Now, to calculate the areas of such figures we first identify the plane figure in combination and then find their area.

Example: Find the area of the shaded design in the given figure, where ABCD is a square of side 10 cm and semi-circles are drawn with each of the sides as diameter. [Take, $\pi = 3.14$]

(REFERENCE: NCERT)





Let the four unshaded regions be marked as 1, 2, 3, 4
We know, area of square = $(\text{side})^2$

Side of square = 10 cm

Area of square ABCD = $(10\text{ cm})^2 = 100\text{ cm}^2$

Area of a semicircle = $\frac{\pi r^2}{2} = \frac{1}{2} X \pi X (5)^2$

($\because$ r = $\frac{d}{2} = \frac{10}{2} = 5cm$

= $\frac{1}{2} X 3.14 X (5)^2 = 39.25 cm^2$

Area of 2 semicircles = $2 \times 39.25 = 78.50\text{ cm}^2$

Area of 1 + Area of 3

= Area of square ABCD - Area of 2 semicircles of radius 5 cm

= $100\text{ cm}^2 - 78.50\text{ cm}^2 = 21.50\text{ cm}^2$

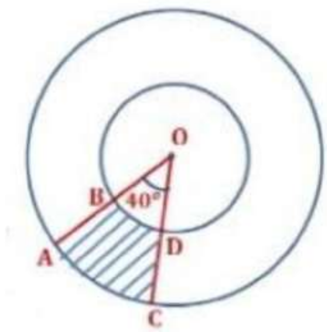
Similarly, Area of 2 + Area of 4 = 21.50 cm^2

$\therefore$ The area of the shaded region

= Area of ABCD – Area of (1 + 2 + 3 + 4)

= $(100 - 2 \times 21.50) = 57\text{ cm}^2$

Example: Find the area of the shaded region in the figure, if radii of the two concentric circles with center O are 21 cm and 14 cm respectively and $\angle AOC = 40^\circ$



OB = 7 cm = r_1 and OA = 14 cm = r_2

$\angle BOD = \angle AOC = 40^\circ$

Area of sector BOD = $\frac{\theta}{360} X \pi r_1^2 = \frac{40}{360} X \pi (14)^2 cm^2$

Area of sector AOC = $\frac{\theta}{360} X \pi {r_2}^2 = \frac{40}{360} X \pi (21)^2 cm^2$

Area of the shaded portion

= Area of sector AOC - Area of sector BOD

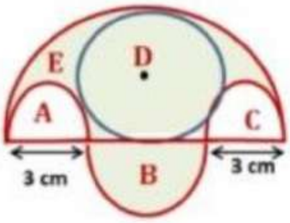
$= \frac{40}{360} X \pi (21)^2 cm^2 - \frac{40}{360} X \pi (14)^2 cm^2 = \frac{40}{360} \pi (21^2 - 14^2)$

$\frac{1}{9} X \frac{22}{7} X (21 + 14)(21 - 14)$ $\{ \because a^2 - b^2 = (a + b)(a - b) \}$

$= \frac{1}{9} X \frac{22}{7} X 35 X 7 = \frac{770}{9} cm^2$

Example: There are three semi-circles A, B, and C having diameter 3 cm each and another semi-circle E having a circle D with diameter 4.5 cm as shown in the figure given below. Find the area of the shaded region.

(REFERENCE: NCERT)



Diameter of semi-circle E = 3 cm + 3 cm + 3 cm = 9 cm

∴ Radius of semi-circle E = $\frac{9}{2}$ cm

Area of semi-circle E = $\frac{1}{2} X \pi X r^2 = \frac{1}{2} X \pi X (\frac{9}{2})^2$

$= \frac{\pi X 81}{2 X 4} = \frac{81\pi}{8} cm^2$

Area of semi-circle A = $\frac{1}{2} X \pi X r^2 = \frac{1}{2} X \pi X (\frac{3}{2})^2$

((∴ Radius of semi-circle E = $\frac{3}{2}$cm)

$= \frac{\pi X 9}{2 X 4} = \frac{9\pi}{8} cm^2$

Similarly, area of semi-circle C = $\frac{9\pi}{8} cm^2$

Area of circle D = $\pi \times r^2 = \pi \times (\frac{4.5}{2})^2 = \frac{20.25\pi}{4} cm^2$

(∴ Radius of circle D = $\frac{4.5}{2}$ cm)

Area of the shaded region

= Area of semi-circle E + Area of semi-circle B – [Area of circle D + Area of semi-circle A + Area of semi-circle C]

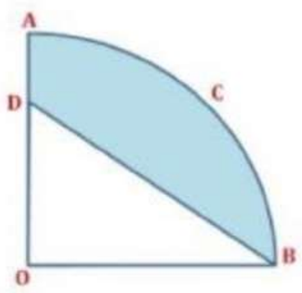
$= \frac{81\pi}{8} + \frac{9\pi}{8} - (\frac{20.25\pi}{4} + \frac{9\pi}{8} + \frac{9\pi}{8}) = \frac{81\pi}{8} - \frac{20.25\pi}{4} - \frac{9\pi}{8} = \frac{81\pi - 40.5\pi - 9\pi}{8}$

$\frac{31.5\pi}{8} = \frac{315\pi}{8 X 10} = \frac{63\pi}{16} cm^2$

Example: In the given figure, OACB is a quadrant of a circle with center O and radius 2.8 cm. If OD = 2 cm, then find the area of the

i) quadrant OACB

ii) shaded region



i) Area of the quadrant OACB = $\frac{1}{4}$× Area of the circle

$= \frac{1}{4} X \pi X r^2 = \frac{1}{4} X \frac{22}{7} x (2.8)^2$

(∵ Radius of the quadrant = 2.8 cm)

Area of the quadrant OACB = 6. 16 cm^2

ii) Area of Δ BOD = $\frac{1}{2} XOBXOD = \frac{1}{2} X 2.8 X 2$

(OB = radius of the circle = 2. 8 cm)

Area of Δ BOD = 2. 8 cm^2

Area of the shaded portion

= Area of the quadrant OACB - Area of Δ BOD

= 6. 16 cm^2 − 2. 8 cm^2

= 3.36 cm^2